**Necessary Formulas:**

1. Entropy, E= –∑ pi log2 pi |;i = 1 to k, where k = number of classes.
2. Average Entropy, ENew = ∑ wi Ei |;i = 1 to n, where n = number of unique values for an attribute.
3. Information Gain, Ig = EStart - ENew

**Iteration 1 (For Selecting the Root Node)**

We have 3 classes. So, The Value of Initial Entropy, EStart will be:

***EStart = – p1 log2 p1 – p2 log2 p2 – p3 log2 p3***

There are 4 instances with classification 1, 5 instances with classification 2 and 15 instances with classification 3. So, p1 = (4/24), p2 = (5/24) and p3 = (15/24).

***EStart = –*** ***(4/24)log2 (4/24)– (5/24)log2 (5/24)– (15/24)log2 (15/24)***

***= 0.4308 + 0.4715 + 0.4238***

***= 1.3261 bits***

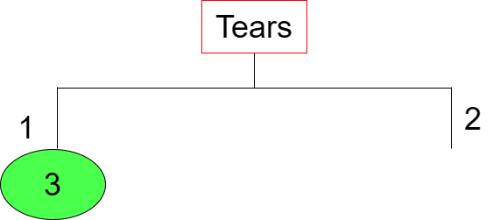
Now, we need to calculate ENew for each of the attributes.

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| **Calculating Entropy for different Values of Age** | | | | |
| |  |  |  | | --- | --- | --- | | For Age = 1, | | | | E1 | = | − (2/8) log2 (2/8)  − (2/8) log2 (2/8)  − (4/8) log2 (4/8) | |  | = | 0.5 + 0.5 +0.5 | |  | = | 1.5 | |  | |  |  |  | | --- | --- | --- | | For Age = 2, | | | | E2 | = | *−* (1*/*8) log2 (1*/*8)  *−* (2*/*8) log2 (2*/*8)  *−* (5*/*8) log2 (5*/*8) | |  | = | 0.375 + 0.5 +0.4238 | |  | = | 1.2988 | |  | |  |  |  | | --- | --- | --- | | For Age = 3, | | | | E3 | = | *−* (1*/*8) log2 (1*/*8)  *−* (1*/*8) log2 (1*/*8)  *−* (6*/*8) log2 (6*/*8) | |  | = | 0.375 + 0.375 + 0.3113 | |  | = | 1.0613 | |
| ENew (Age)= (8/24) E1 + (8/24) E2 + (8/24) E3 = 1*.*2867 bits  Information Gain, Ig (Age)= *EStart -* ENew (Age)= 1.3261 – 1.2867= 0.0394 bits | | | | |

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| **Calculating Entropy for different Values of SpecRx** | | |
| |  |  |  | | --- | --- | --- | | For SpecRx = 1, | | | | E1 | = | − (3/12) log2 (3/12)  − (2/12) log2 (2/12)  − (7/12) log2 (7/12) | |  | = | 0.5 + 0.4308 + 0.4536 | |  | = | 1.3844 | |  | |  |  |  | | --- | --- | --- | | For SpecRx = 2, | | | | E2 | = | − (1/12) log2 (1/12)  − (3/12) log2 (3/12)  − (8/12) log2 (8/12) | |  | = | 0.2988 + 0.5 + 0.3900 | |  | = | 1.1887 | |
| ENew (SpecRx) = (12/24) E1 + (12/24) E2 = 1.2866 bits  Information Gain, Ig (SpecRx) = *EStart -* ENew (SpecRx)= 0.0395 bits | | |

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| **Calculating Entropy for different Values of Astig** | | |
| |  |  |  | | --- | --- | --- | | For Astig = 1, | | | | E1 | = | − (0/12) log2 (0/12)  − (5/12) log2 (5/12)  − (7/12) log2 (7/12) | |  | = | 0 + 0.5263 + 0.4536 | |  | = | 0.9799 | |  | |  |  |  | | --- | --- | --- | | For Astig = 2, | | | | E2 | = | − (4/12) log2 (4/12)  − (0/12) log2 (0/12)  − (8/12) log2 (8/12) | |  | = | 0.5283 + 0 + 0.3900 | |  | = | 0.9183 | |
| ENew (Astig) = (12/24) E1 + (12/24) E2 = 0.9491 bits  Information Gain, Ig (Astig) = *EStart -* ENew (Astig)= 0.377 bits | | |

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| **Calculating Entropy for different Values of Tears** | | |
| |  |  |  | | --- | --- | --- | | For Tears = 1, | | | | E1 | = | − (0/12) log2 (0/12)  − (0/12) log2 (0/12)  − (12/12) log2 (12/12) | |  | = | 0 + 0 + 0 | |  | = | 0 | |  | |  |  |  | | --- | --- | --- | | For Tears = 2, | | | | E2 | = | − (4/12) log2 (4/12)  − (5/12) log2 (5/12)  − (3/12) log2 (3/12) | |  | = | 0.5283 + 0.5263 + 0.5 | |  | = | 1.5546 | |
| ENew (Tears) = (12/24) E1 + (12/24) E2 = 0.7773 bits  Information Gain, Ig (Tears) = *EStart -* ENew (Tears)= **0.5488 bits** | | |



**Iteration 2 (For Branch Tears = 2)**

There are 4 instances with classification 1, 5 instances with classification 2 and 3 instances with classification 3. So, p1 = (4/12), p2 = (5/12) and p3 = (3/12).

***EStart = – (4/12)log2 (4/12)– (5/12)log2 (5/12)– (3/12)log2 (3/12)***

***= 0.5283 + 0.5263 + 0.5***

***= 1.5546 bits***

Now, we need to calculate ENew for each of the attributes.

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| **Calculating Entropy for different Values of Age** | | | | |
| |  |  |  | | --- | --- | --- | | For Age = 1, | | | | E1 | = | − (2/4) log2 (2/4)  − (2/4) log2 (2/4)  − (0/4) log2 (0/4) | |  | = | 0.5 + 0.5 + 0 | |  | = | 1.0 | |  | |  |  |  | | --- | --- | --- | | For Age = 2, | | | | E2 | = | − (1/4) log2 (1/4)  − (2/4) log2 (2/4)  − (1/4) log2 (1/4) | |  | = | 0.5 + 0.5 + 0.5 | |  | = | 1.5 | |  | |  |  |  | | --- | --- | --- | | For Age = 3, | | | | E3 | = | − (1/4) log2 (1/4)  − (1/4) log2 (1/4)  − (2/4) log2 (2/4) | |  | = | 0.5 + 0.5 +0.5 | |  | = | 1.5 | |
| ENew (Age)= (4/12) E1 + (4/12) E2 + (4/12) E3 = 1*.*3333 bits  Information Gain, Ig (Age)= *EStart -* ENew (Age)= 1.5546 – 1.3333= 0.2213 bits | | | | |

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| **Calculating Entropy for different Values of SpecRx** | | |
| |  |  |  | | --- | --- | --- | | For SpecRx = 1, | | | | E1 | = | − (3/6) log2 (3/6)  − (2/6) log2 (2/6)  − (1/6) log2 (1/6) | |  | = | 0.5 + 0.5283 + 0.4308 | |  | = | 1.4591 | |  | |  |  |  | | --- | --- | --- | | For SpecRx = 2, | | | | E2 | = | − (1/6) log2 (1/6)  − (3/6) log2 (3/6)  − (2/6) log2 (2/6) | |  | = | 0.4308 + 0.5 + 0.5283 | |  | = | 1.4591 | |
| ENew (SpecRx) = (6/12) E1 + (6/12) E2 = 1.4591 bits  Information Gain, Ig (SpecRx) = *EStart -* ENew (SpecRx)= 0.096 bits | | |

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| **Calculating Entropy for different Values of Astig** | | |
| |  |  |  | | --- | --- | --- | | For Astig = 1, | | | | E1 | = | − (0/6) log2 (0/6)  − (5/6) log2 (5/6)  − (1/6) log2 (1/6) | |  | = | 0 + 0.2192 + 0.4308 | |  | = | 0.6500 | |  | |  |  |  | | --- | --- | --- | | For Astig = 2, | | | | E2 | = | − (4/6) log2 (4/6)  − (0/6) log2 (0/6)  − (2/6) log2 (2/6) | |  | = | 0.3900 + 0 + 0.5283 | |  | = | 0.9183 | |
| ENew (Astig) = (6/12) E1 + (6/12) E2 = 0.7842 bits  Information Gain, Ig (Astig) = *EStart -* ENew (Astig)= **0.7704 bits** | | |



**Iteration 3 (For Branch Astig = 1)**

There are 5 instances with classification 2 and 1 instance with classification 3. So, p1 = (5/6) and p2 = (1/6).

***EStart = – (5/6)log2 (5/6)– (1/6)log2 (1/6)***

***= 0.2192 + 0.4308***

***= 0.65 bits***

Now, we need to calculate ENew for each of the attributes.

|  |  |  |  |  |
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| **Calculating Entropy for different Values of Age** | | | | |
| |  |  |  | | --- | --- | --- | | For Age = 1, | | | | E1 | = | − (0/2) log2 (0/2)  − (2/2) log2 (2/2)  − (0/2) log2 (0/2) | |  | = | 0 + 0 + 0 | |  | = | 0 | |  | |  |  |  | | --- | --- | --- | | For Age = 2, | | | | E2 | = | − (0/2) log2 (0/2)  − (2/2) log2 (2/2)  − (0/2) log2 (0/2) | |  | = | 0 + 0 + 0 | |  | = | 0 | |  | |  |  |  | | --- | --- | --- | | For Age = 3, | | | | E3 | = | − (0/2) log2 (0/2)  − (1/2) log2 (1/2)  − (1/2) log2 (1/2) | |  | = | 0 + 0.5 + 0.5 | |  | = | 1 | |
| ENew (Age)= (2/6) E1 + (2/6) E2 + (2/6) E3 = 0.3333 bits  Information Gain, Ig (Age)= *EStart -* ENew (Age)= 0.6500 – 0.3333= **0.3167 bits** | | | | |

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| **Calculating Entropy for different Values of SpecRx** | | |
| |  |  |  | | --- | --- | --- | | For SpecRx = 1, | | | | E1 | = | − (0/3) log2 (0/3)  − (2/3) log2 (2/3)  − (1/3) log2 (1/3) | |  | = | 0 + 0.3900 + 0.5283 | |  | = | 0.9183 | |  | |  |  |  | | --- | --- | --- | | For SpecRx = 2, | | | | E2 | = | − (0/3) log2 (0/3)  − (3/3) log2 (3/3)  − (0/3) log2 (0/3) | |  | = | 0 + 0 + 0 | |  | = | 0 | |
| ENew (SpecRx) = (3/6) E1 + (3/6) E2 = 0.4592 bits  Information Gain, Ig (SpecRx) = *EStart -* ENew (SpecRx)= 0.1908 bits | | |

A diagram of a number of people

AI-generated content may be incorrect.

**Iteration 4 (For Branch Astig = 2)**

There are 4 instances with classification 1 and 2 instance with classification 3. So, p1 = (4/6) and p2 = (2/6).

***EStart = – (4/6)log2 (4/6)– (2/6)log2 (2/6)***

***= 0.3900 + 0.5283***

***= 0.9183 bits***

Now, we need to calculate ENew for each of the attributes.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Calculating Entropy for different Values of Age** | | | | |
| |  |  |  | | --- | --- | --- | | For Age = 1, | | | | E1 | = | − (2/2) log2 (2/2)  − (0/2) log2 (0/2)  − (0/2) log2 (0/2) | |  | = | 0 + 0 + 0 | |  | = | 0 | |  | |  |  |  | | --- | --- | --- | | For Age = 2, | | | | E2 | = | − (1/2) log2 (1/2)  − (0/2) log2 (0/2)  − (1/2) log2 (1/2) | |  | = | 0.5 + 0 + 0.5 | |  | = | 1 | |  | |  |  |  | | --- | --- | --- | | For Age = 3, | | | | E3 | = | − (1/2) log2 (1/2)  − (0/2) log2 (0/2)  − (1/2) log2 (1/2) | |  | = | 0.5 + 0 + 0.5 | |  | = | 1 | |
| ENew (Age)= (2/6) E1 + (2/6) E2 + (2/6) E3 = 0.6667 bits  Information Gain, Ig (Age)= *EStart -* ENew (Age)= 0.9183 – 0.6667= 0.2516 bits | | | | |

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| **Calculating Entropy for different Values of SpecRx** | | |
| |  |  |  | | --- | --- | --- | | For SpecRx = 1, | | | | E1 | = | − (3/3) log2 (3/3)  − (0/3) log2 (0/3)  − (0/3) log2 (0/3) | |  | = | 0 + 0 + 0 | |  | = | 0 | |  | |  |  |  | | --- | --- | --- | | For SpecRx = 2, | | | | E2 | = | − (1/3) log2 (1/3)  − (0/3) log2 (0/3)  − (2/3) log2 (2/3) | |  | = | 0.5283 + 0 + 0.3900 | |  | = | 0.9183 | |
| ENew (SpecRx) = (3/6) E1 + (3/6) E2 = 0.4592 bits  Information Gain, Ig (SpecRx) = *EStart -* ENew (SpecRx)= **0.4591 bits** | | |

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AI-generated content may be incorrect.